## *ON PROBLEM* 132.

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THE integral required in this question is a special case of the one discussed by Legendre in his treatise on elliptic functions, Chap. XXVI. p. 165. Legendre shows that the elliptic integrals disappear, and that the result can be expressed in logarithmic and circular functions. His method is complicated however, and Clausen has obtained the result by the following simple substitutions. Let

$$z=\frac{x-1}{\sqrt{(x^3-1)}}:\qquad z'=\sqrt{(x^3-1)}:\qquad z''=\frac{(x-1)^2}{\sqrt{(x^3-1)}}:$$

Then

$$dz=-\frac{x^3-3x^2+2}{2\sqrt{(x^3-1)^3}}dx:\qquad dz'=\frac{3}{2}.\frac{x^2.dx}{\sqrt{(x^3-1)}}:$$

$$dz''=\frac{1}{2}.\frac{x^4+2x^3-3x^2-4x+4}{\sqrt{(x^3-1)^3}}dx:$$

Hence

$$\frac{dz}{1-3z^2}=-\frac{1}{2}.\frac{x^2-2x-2}{x^2-2x+4}.\frac{dx}{\sqrt{(x^3-1)}}:\quad \frac{dz'}{z'^2+9}=\frac{3}{2}.\frac{x^2}{x^3+8}.\frac{dx}{\sqrt{(x^3-1)}}:$$

$$\frac{dz''}{z''^2+9}=\frac{1}{2}.\frac{(x-1)(x^3+3x^2-4)}{(x+2)(x^3+3x^2-4)}.\frac{dx}{\sqrt{(x^3-1)}}=\frac{1}{2}.\frac{x-1}{x+2}.\frac{dx}{\sqrt{(x^3-1)}}:$$

$$\therefore\qquad \frac{dz}{1-3z^2}+\frac{dz'}{z'^2+9}+\frac{dz''}{z''^2+9}=\frac{bx.dx}{(x^3+8)\sqrt{(x^3-1)}}:$$

The integrations can now be performed and we have

$$\int\frac{xdx}{(x^3+8)\sqrt{(x^3-1)}}=\frac{1}{12\sqrt{3}}.\log\left(\frac{1+z\sqrt{3}}{1-z\sqrt{3}}\right)+\frac{1}{18}\tan^{-1}\frac{z'}{3}+\frac{1}{18}\tan^{-1}\frac{z''}{3}$$

$$=\frac{1}{12\sqrt{3}}.\log\frac{\sqrt{(x^2+x+1)}+\sqrt{(x-1)}.\sqrt{3}}{\sqrt{(x^2+x+1)}-\sqrt{(x-1)}.\sqrt{3}}+\frac{1}{18}\tan^{-1}\frac{3x(x-1)}{(4-x)(x^3-1)^{\frac{1}{2}}}.$$

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REMARKS BY PROF. ORSON PRATT, SEN., SALT LAKE CITY, UTAH.—THE six propositions, published in the ANALYST, Vol. III, No. 6, and demonstrated in the last No., are not "presented as the basis of a theory of gravitation." They have no bearing on or reference to the *cause* of the force called gravity, or to the exploded hypothesis of an ethereal pressure, exerted in the direction of a gravitating center.